From 1976 to 1978 I sent a copy of my paper to many famous mathematicians. It is now 2023, forty-five years later, below are the responses I can find in 2023. The world's best mathematicians could not find any errors in my paper.

Before you read my paper you should read what was said about it by some of the world's best mathematicians. Therefore, my paper comes up after the following letters.

The below letter from Professor Gian-Carlo Rota of the Massachusetts Institute of Technology (MIT) indicates that he studied my paper and found no errors. As seen from his letter, I mailed my paper to him on September 9, 1976, he kept my paper for close to 5 weeks and then he personally replied to me on October 13, 1976. If MIT and the journal *"Advances in Mathematics"* was simply going to send me a form rejection letter that would have been done by one of Professor Rota's assistants sometime in September of 1976.

ADVANCES IN MATHEMATICS

Editor:
GIAN-CARLO ROTA
Department of Mathematics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Publishers:
ACADEMIC PRESS, INC.
111 Fifth Avenue
New York, N. Y. 10003

October 13, 1976

Mr. Shawn D. Olfman
70 Polson Avenue
Winnipeg, Manitoba
R2W-OM2 Canada

Dear Mr. Olfman:

Thank you for your letter of Sept. 9, 1976 and paper entitled "Useful thoughts on Infinity". I very much regret that I am unable to publish your manuscript in Advances as the backlog for this journal now runs to five years. We have been forced to return many newly submitted papers in order to decrease the backlog.

Sincerely,

Gian-Carlo Rota

GCR:lb
encl.

The next letter shows that I wrote to a professor at Yale University. (In the 1970's there was no Internet, information was from journals and people working in the field; the information I was able to obtain was a few years old, which back in the 70's was current enough when it came to getting a person's Who's Who credentials and address.) His wife responded from the Yale Department of Mathematics and directed me to a noted mathematician, Professor Luxemburg, at the California Institute of Technology (Caltech). The Yale math department had my paper for several months (and found no errors in my paper) before recommending that I contact Professor Luxemburg at Caltech; if Yale had found errors in my paper they would have simply told that to me and they would not have recommended that I contact Professor Luxemburg.

(The math on infinity created by Georg Cantor had been taught and studied since about the turn of the 20th century, if my paper *"Useful Thoughts on Infinity"* was correct, then Cantor's teachings on infinity were wrong, which meant that Cantor's work, which had been studied and taught for about 75 years, and which was included in the curriculums, text books and papers of every university in the world, was wrong, and that would be a huge embarrassment for the math world, including for the professors who were studying my paper; everything on infinity and on infinitesimals would have to be redone and re-taught according to my paper (the paper of an undergrad student at a public university in Canada).

Yale wasn't telling me to write to Professor Luxemburg at Caltech to prank Professor Luxemburg, they were hoping that Professor Luxemburg could find an error in my work.

YALE UNIVERSITY
DEPARTMENT OF MATHEMATICS
BOX 2155 · YALE STATION
NEW HAVEN · CONNECTICUT 06520

July 8th, 1977

Dear Mr. Olfman,

The letter you wrote to my husband reached me to-day. My husband died in April 1974. I therefore refer you to Prof. A.W. LUXEMBURG, chairman of Math. at CALTECH PASADENA, CALIF. 91125. He would be the right person to help you, I think.

Yours, sincerely,

R. Robinson

By the end of July of 1977 I had sent a copy of my paper *"Useful Thoughts on Infinity"* to Professor Luxemburg at Caltech, he could not find any errors in my paper and advised me that he had passed it on to Professor Stroyan (who was also a math professor at Caltech). Professor Stroyan advised me that he was studying my paper and would respond to me as soon as he had finished studying it; months went by and on November 25, 1977 I again wrote to Professor Luxemburg, who responded to me on December 7, 1977, advising me that Professor Stroyan was still studying my paper and that he would respond to me as soon as he finished studying my paper.

CALIFORNIA INSTITUTE OF TECHNOLOGY

ALFRED P. SLOAN LABORATORY OF MATHEMATICS AND PHYSICS
PASADENA, CALIFORNIA 91125

MATHEMATICS 253-37 TELEPHONE (213) 795-6811

December 7, 1977

Mr. Shawn Olfman
70 Polson Avenue
Winnipeg, Manitoba
R2W OM2
Canada

Dear Mr. Olfman:

This is to answer your letter of November 25, 1977.

I am sorry that you did not hear again from Professor Stroyan. I am sure that he will contact you as soon as he has finished studying your paper.

Sincerely yours,

W. A. J. Luxemburg

W. A. J. Luxemburg

WAJL:rs

I heard nothing further from Caltech; they knew that my paper was correct and that the math on infinity which had been taught since about the beginning of the 20th century was wrong, but they weren't about to take the information to the world math community that an undergrad student at a public university in Canada had proven the last about 75 years of math on infinity to be wrong, and the books and courses now had to be re-written.

Between September of 1976 and (if Professor Luxemburg was correct in his December 7, 1977 letter, where he wrote that Professor Stroyan was continuing to study my paper) some time into 1978 or beyond, my paper *"Useful Thoughts on Infinity"* was studied by some of the greatest math minds in the world, and they found no errors in my work.

In the mid to late 1970's *"The Journal of Irreproducible Results"* was a well respected journal in the scientific Ph.D. community in general, it published facetious or sometimes sarcastic articles, often of supposed results (which were erroneous) for the intellectual entertainment of the general scientific community. I thought that I could get my paper into that journal even though my paper was a serious paper, because it showed the current teachings on infinity to be wrong; I also thought it possible that they might not notice that my paper was a serious paper which would completely change the mathematical world's concept of infinity. They ultimately responded **that my paper was excellent**, but that most of their readers would not understand it and therefore they were not going to publish my paper. It is correct to say that to understand the math in my paper a person would have to have a sophisticated knowledge in a specialized area of math.

Possibly their stated reason for not publishing my paper was their reason, possibly they didn't want to publish a paper that would show that the math world had been wrong for at least the last seventy-five years on their teachings of infinity.

The bottom line is that the publisher of that journal felt that he could not just not publish my paper without also telling me that my paper was excellent.

THE JOURNAL OF
Irreproducible Results
Inc. ®

Box 234 Chicago Heights, Illinois 60411 U.S.A. (312) 563-1770

April 21, 1978

Shawn D. Olfman
70 Polson Avenue
Winnipeg, Manitoba,
Canada R2W-0M2

Dear Mr. Olfman:

We would like very much to submit your paper "Useful Thoughts on Infinity" for review by our referees, but find that some of the text is handwritten, all of it is xeroxed, and a lot of it contains material which we would have difficulty typesetting and would require original art and a clean type-written copy.

If you can submit the manuscript in a form suitable for review in accord with the above remarks, we would very much appreciate it and would be very happy to consider it for publication.

Sincerely yours,

THE JOURNAL OF IRREPRODUCIBLE RESULTS, INC.

George H. Scherr, Ph.D.
Publisher

GHS;ss

Dear Dr. Scherr;
I have redone the paper as you requested, as it takes a long time to redo my paper I would appreciate its return if you decide not to use it.
I am leaving the continent until Sept 5, therefore if further communication is necessary please delay it until then. —[illegible]

Sincerely yours.
[illegible]

THE JOURNAL OF
Irreproducible Results
Inc. ®
Box 234 Chicago Heights, Illinois 60411 U.S.A. (312) 563-1770

September 7, 1978

Mr. Shawn D. Olfman
70 Polson Avenue
Winnipeg, Manitoba,
Canada R2W-OM2

Dear Mr. Olfman:

We are returning your paper "Useful Thoughts on Infinity", which we regret has not been accepted for publication in our Journal.

We have uniformly followed the procedure in the past of not presenting comments of reviewers in explanation as to why the paper was rejected because we are not a technical journal and the criteria for acceptance may have little relationship to the merit of the technical acumen and more with the way in which the paper can fasinate readers especially outside the domain that the writer is working in. This is a particularly important point to make because I think your paper is an excellent one and would have interested many of our readers, but the great majority would frankly, at least in our opinion, have lost the point.

Thank you for your interest in writing.

Sincerely yours,

THE JOURNAL OF IRREPRODUCIBLE RESULTS, INC.

George H. Scherr

George H. Scherr, Ph.D.
Publisher

GHS:ss

Useful Thoughts on Infinity

Written By: Shawn David Olfman

Completed: July 10, 1976

Written By: Shawn D. Olfman
70 Polson Ave.
Winnipeg, Manitoba
Canada R2W 0M2

Hebrew letters used:

 lamed

 aleph

 capital aleph

 capital lamed

 sadhe

 capital Pe

The purpose of this paper is two-fold. Firstly,to explain my concept of infinity and how infinite numbers should be expressed. Secondly, to present a number system which is a consequence of my infinite numbers.

There are two ideas upon which this entire paper rests. The first is my definition of what infinity is. The second, is that there is an end to the real number line*1 (this statement is also given the status of a definition). Before beginning the mathematics of this paper, convince yourself that these two ideas are correct, for the mathematics is logically built out of them.

The dictionary definition of infinity tells us that infinity is the quality of being boundless; an infinite number is one which is indefinitely great. This definition, however, offers us no means of writing down a workable infinite number (ie. an infinite number which can be used in calculations in other then a symbolic sense). Hence I put forth the following definition of infinity: Definition: Infinity is that place or object which can be travelled towards but never reached. From which I may draw the conclusion that, either infinity is stationary and no-where,*2 or it is moving and it is somewhere. I choose the latter conclusion for to say infinity is no-where is to say it does not exist, for all that exists must exist some-where.

In the later conslusion, that infinity is moving and is some-where are of necessity two implicit additional statements. Firstly, that

*1 This does not mean that there is an end to the real numbers. What it means is that they must be represented by something that has an end.

*2 I say it is no-where if it is stationary because any object or place which is somewhere and not moving can be reached.

all things moving toward infinity are not at infinity, and secondly that infinity is moving away from all things moving toward it and infinity is not moving slower then all things moving toward it.

Realizing that the real numbers can be expressed as stationary points on the real number line, I propose the following definition of an infinite number.[*3] An infinite number is represented by a point which is initially beyond the real number line and is moving away from its end of the real number line. In keeping with this definition I propose the following model of an infinite number.

Symbols used: ל pronounced lamed (a letter of the Hebrew alphabet)

א pronounced aleph (a letter of the Hebrew alphabet)

Definition: Let $\mathbb{R}^+$ be a metric space and let $\mathbb{R}^+ = (\{\mathbb{R}, ל(n,y,\tau)\}, א)$

$\mathbb{R}$ = set of all reals

Definition: Let $ל(n,y,\tau)$ be a point in $\mathbb{R}^+$ which is, at $\tau = 0$, $|n|$ units beyond the real number line on the right side of zero if the sign of n is positive, and $|n|$ units beyond the real number line on the left side of zero if the sign of n is negative; and moving away from its end of the real number line at y units / units of τ, beginning at $\tau = 0$, where $n, y, \tau \in \mathbb{R}$, $\tau \geq 0$, the sign of y is the same as the sign of n.[*4*5] Thus $ל(n,y,\tau)$ is an infinite number whenever $y \neq 0$.

*3 Read Appendix I and then read this introduction over again before reading the mathematics .

*4 τ represents the "time" at which we are looking at the infinite number; as such its value is never written down, why, shall become clear as you read this paper.

*5 I like to think of the infinite number as being $|n| + |y \cdot \tau|$ units beyond the respective end of the real number line at time τ .

Aside: In a casual sense y may be thought of as the speed with which $\zeta(n,y,\tau)$ is receeding from its respective end of the real number line.

Definition: Let the metric for $\mathbb{R}^+$ be denoted by lc.

$$\mathrm{lc}(a,b) = |a-b| \quad \forall a,b \in \mathbb{R}$$

$$\mathrm{lc}(a, \zeta(n,y,\tau)) = \mathrm{lc}(\zeta(n,y,\tau), a) \quad \forall a \in \mathbb{R}$$

$$\mathrm{lc}(a,\zeta(n,y,\tau)) = \left.\begin{array}{ll} \zeta(|n-a|, |y|, \tau) & \text{if } |n|-|a| \geq 0 \\ \left.\begin{array}{ll} = \zeta\left(0, \frac{|y|}{2}, \tau\right) & \text{for } \tau \leq \left|\frac{2\cdot(|a|-|n|)}{y}\right| \\ = \zeta\left(0, \left||y| - \frac{|a|-|n|}{\tau}\right|, \tau\right) & \text{for } \tau > \left|\frac{2\cdot(|a|-|n|)}{y}\right| \end{array}\right\} & \text{if } |n|-|a|<0 \end{array}\right\} \text{if } n\geq 0, a \geq 0 \text{ or } n<0, a<0$$

$$= \zeta(|n-a|, |y|, \tau) \quad \text{if } a<0, n\geq 0 \text{ or } a\geq 0, n<0$$

$$\forall a \in \mathbb{R}, y \neq 0$$

$$\mathrm{lc}(\zeta(n_1, y_1, \tau_1), \zeta(n_0, y_0, \tau_0)) = 0 \quad \text{if } n_1 = n_0, y_1 = y_0, \tau_1 = \tau_0$$

$$\text{is undefined if } n_1 \neq n_0 \text{ or } y_1 \neq y_0 \text{ or } \tau_1 \neq \tau_0$$

(Note that each time we write down the elements of $\mathbb{R}^+$, $\zeta(n,y,\tau)$ may be different, however, there can be only one $\zeta(n,y,\tau)$ in $\mathbb{R}^+$.)

It will now be instructive to work out examples illustrating distances between real numbers, ie. elements of $\mathbb{R}$, and an infinite number.

eg.1 $lc\left(6, \text{ㄅ}(10, 3, \tau)\right) = \text{ㄅ}\left(|10-6|, 3, \tau\right) = \text{ㄅ}(4, 3, \tau)$

eg.2 $lc\left(-2, \text{ㄅ}(1, 6, \tau)\right) = \text{ㄅ}(3, 6, \tau)$

eg.3 $lc\left(4, \text{ㄅ}(1, 1, \tau)\right) = \text{ㄅ}(0, \tfrac{1}{2}, \tau)$ for $\tau \leq 6$

$= \text{ㄅ}\left(0, (1 - \tfrac{3}{\tau}), \tau\right)$ for $\tau > 6$

eg.4 $lc\left(-6, \text{ㄅ}(-2, -1, \tau)\right) = \text{ㄅ}(0, \tfrac{1}{2}, \tau)$ for $\tau \leq 8$

$= \text{ㄅ}\left(0, |1 - \tfrac{4}{\tau}|, \tau\right)$ for $\tau > 8$

Having illustrated that the distance between real numbers and an infinite number are infinite but different; it is now reasonable to define multiplication, subtraction and addition between real numbers and an infinite number, division shall be discussed at the end of the paper. (Again note that opperations between different infinite numbers are impossible, because there is only one infinite number in $\mathbb{R}^+$).

Addition will be defined as follows: $\forall a \in \mathbb{R}$, $y \neq 0$

If the sign of n is positive:

$$\text{ㄅ}(n, y, \tau) + a = \text{ㄅ}(n+a, y, \tau) \quad \text{if } n+a \geq 0$$

$$\left.\begin{aligned} &= \text{ㄅ}(0, \tfrac{y}{2}, \tau) \quad \text{for } \tau \leq \left|\frac{2\cdot(|a|-|n|)}{y}\right| \\ &= \text{ㄅ}\left(0, y - \frac{|a|-|n|}{\tau}, \tau\right) \quad \text{for } \tau > \left|\frac{2\cdot(|a|-|n|)}{y}\right| \end{aligned}\right\} \text{if } n+a < 0$$

If the sign of n is negative:

$$\text{ϟ}(n,y,\tau)+a = \text{ϟ}(n+a,\, y,\, \tau) \quad \text{if } n+a \le 0$$

$$\left.\begin{aligned} &= \text{ϟ}\left(-0,\, \tfrac{y}{2},\, \tau\right) \quad \text{for } \tau \le \left|\frac{2\cdot(|a|-|n|)}{y}\right| \\ &= \text{ϟ}\left(-0,\, y+\frac{|a|-|n|}{\tau},\, \tau\right) \quad \text{for } \tau > \left|\frac{2\cdot(|a|-|n|)}{y}\right| \end{aligned}\right\} \text{if } n+a>0$$

The following relations will complete addition and define subtraction.

$$\text{ϟ}(n,y,\tau)+a = a+\text{ϟ}(n,y,\tau)$$

$$\text{ϟ}(n,y,\tau)-a = \text{ϟ}(n,y,\tau)+(-a)$$

$$a-\text{ϟ}(n,y,\tau) = a+\text{ϟ}(-n,-y,\tau)$$

Multiplication will be defined as follows: $\forall a \in \mathbb{R} \quad a \ne 0$

$$a \times \text{ϟ}(n,y,\tau) = \text{ϟ}(n,y,\tau)\times a$$

$$\text{ϟ}(n,y,\tau)\times a = \text{ϟ}(n\times a,\, y\times a,\, \tau)$$

The following examples should help illustrate the opperations:

eg. 5 $\text{ϟ}(3,2,\tau)+5 = \text{ϟ}(8,2,\tau)$

eg. 6 $\text{ϟ}(3,2,\tau)-6 = \text{ϟ}(0,1,\tau)$ for $\tau \le 3$

$= \text{ϟ}\left(0,\, 2-\frac{3}{\tau},\, \tau\right)$ for $\tau > 3$

eg. 7 $\text{ϟ}(-2,-1,\tau)+3 = \text{ϟ}\left(-0,\, -\frac{1}{2},\, \tau\right)$ for $\tau \le 2$

$= \text{ϟ}\left(-0,\, \left(-1+\frac{1}{\tau}\right),\, \tau\right)$ for $\tau > 2$

eg.8 $\mathfrak{h}(3, \frac{1}{2}, \tau) \times 6 = \mathfrak{h}(18, 3, \tau)$

eg.9 $\mathfrak{h}(1, 2, \tau) \times -10 = \mathfrak{h}(-10, -20, \tau)$

Having defined an infinite number along with what I like to think of as the fundamental operations on that number the next logical avenue of progression would be to try and assign an infinite number to an infinite set of objects (ie. state the cardinality of various infinite sets). However, I shall not attempt that at this stage, for reasons which will soon become obvious, but shall now define the multiplication of an infinite number by zero.

Definition: $\mathfrak{h}(n, y, \tau) \times 0 \equiv 0 \times \mathfrak{h}(n, y, \tau) = \mathfrak{h}(0, 0, \tau)$

According to the definition of $\mathfrak{h}(n, y, \tau)$, $\mathfrak{h}(0, 0, \tau)$ is a point zero units away from the right end of the real number line, moving away from the right end of the real number line at zero units / units of τ. Hence for all time τ it is the right end of the real number line, and thus the largest real number. (The smallest real number would be $\mathfrak{h}(-0, -0, \tau)$).

We are now faced with the problem of what $\mathfrak{h}(0, 0, \tau) + a$, $\forall a \in \mathbb{R}$ is equal to.

There are two possible solutions, either $\mathfrak{h}(0, 0, \tau_1) + a = \mathfrak{h}(a, 0, \tau_1)$ or $\mathfrak{h}(0, 0, \tau_1) + a = \mathfrak{h}(0, 0, \tau_2)$.

Recalling appendix I you will see that the second solution to the problem $\mathfrak{h}(0, 0, \tau_1) + a$, is saying that because the right end of the real number line is forever expanding no matter what finite amount one adds to the right end of the real number line at a certain time τ_1 : this will correspond to the right end of the real number line plus zero at some other time τ_2 .(The same is true for the left end of the real number line ie. $\mathfrak{h}(-0, -0, \tau)$.) In other words, an end of the real number line will always be an end of the real number line no matter what one adds to it, or subtracts from it (with one exception, $\zeta(0,0,\tau) + \zeta(-0,-0,\tau) = 0$).

Before we can accept the second solution I must prove that it is the correct one.

Proposition: $\zeta(0,0,\tau_1) + a = \zeta(0,0,\tau_2)$

Proof: Given $a,\ \zeta(0,0,\tau_1)\ ,\ \zeta(0,0,\tau_2) \in \mathbb{R}$

Let us deal with the metric space $\mathbb{R}$.

Assume $\zeta(0,0,\tau_1) + a = \zeta(a,0,\tau_1)$. We can see that $\zeta(a,0,\tau_1)$ by definiton is a point which is $|a|$ units away from the right end of the real number line. Without loss of generallity I can let a be greater then zero (ie. $a > 0$), therefore $\zeta(a,0,\tau_1)$ is, for all time τ , a units beyond the right end of the real number line, therefore $\zeta(a,0,\tau_1)$ is not a real number. But one of the axioms of the real numbers states that $\forall a,b \in \mathbb{R}$, $a+b \in \mathbb{R}$ hence a contradiction, thus $\zeta(0,0,\tau_1) + a \neq \zeta(a,0,\tau_1)$.

If on the other hand, $\zeta(0,0,\tau_1) + a = \zeta(0,0,\tau_2)$ then there is no contradiction because $\zeta(0,0,\tau_2) \in \mathbb{R}$. Thus we can conclude that $\zeta(0,0,\tau_1) + a = \zeta(0,0,\tau_2)$ Q.E.D.

You might now be wondering how one goes about determining τ_1, τ_2, etc. The answer is that you don't. The reason for this, is that in mathematics as we now know it there is no use for the time at which a number exists.*6 Therefore, since we don't care what τ_1 and τ_2 are, we may simply write $\zeta(0,0,\tau) + a = \zeta(0,0,\tau)$.

Multiplication of $\zeta(0,0,\tau)$ by any real number including itself was

*6 As a point of interest, the τ's could be determined by assigning a value to the expansion of the real number line and assigning a value to $\zeta(0,0,0)$ (ie. $\zeta(0,0,\tau_0)$ where $\tau_0 = 0$). The difficulty in doing this would lie in determining a value for the expansion of the real number line and a τ_0 such that $\forall a \in \mathbb{R}$, $\zeta(0,0,\tau_x) + a = \zeta(0,0,\tau_y)$.

defined when I defined multiplication of the lameds by a real number. It was defined as follows: $\forall a \in \mathbb{R},\ a \neq 0$

$$ℸ(0,0,\tau) \times a = ℸ(0 \times a, 0 \times a, \tau) = ℸ(0,0,\tau)$$

Multiplication of $ℸ(0,0,\tau)$ by zero is defined as follows:

$$ℸ(0,0,\tau) \times 0 \equiv 0 \times ℸ(0,0,\tau) \equiv 0$$

The reason for this is that $ℸ(0,0,\tau)$ is real and any real number multiplied by zero is zero.

The remainder of this paper deals with the conclusions that can be drawn from my ideas, to derive them was the reason I wrote this paper; therefore I urge the reader to consiously note the differences between what I say and what present mathematics says.

As stated in the beginning of this paper, I am going to introduce a new number system. Once the following definitions and theorms have been understood, I will be able to introduce this system.

Symbol used: ℶ pronounced lamed (the capitol Hebrew letter)

Definition: $\frac{0}{0} \equiv 0$

Definition: $\forall b \in \mathbb{R} \quad \frac{b}{0} \equiv ℸ(0,0,\tau)$

Definition: $ℶ \equiv ℸ\big(ℸ(0,0,\tau), ℸ(0,0,\tau), \tau\big)$

Theorem: ℶ Is the largest infinite number

Proof: All infinite numbers are of the form $ℸ(n,y,\tau)$ where $n,y \in \mathbb{R}$

However $ℸ(0,0,\tau) \in \mathbb{R}$ and $\forall b \in \mathbb{R} \quad ℸ(0,0,\tau) \geq b$

Q.E.D.

Theorem: $\forall n,y \in \mathbb{R} \ni y \neq 0 \quad \frac{ℸ(n,y,\tau)}{0} = ℶ$

Proof: $\frac{ℸ(n,y,\tau)}{0} = \frac{ℸ(n,y,\tau)}{1} \times \frac{1}{0} = ℸ\left(\frac{n}{0}, \frac{y}{0}, \tau\right)$

but $\frac{n}{0} = \frac{y}{0} = ℸ(0,0,\tau)$ Q.E.D.

Note that this also states that ף/0 = ף

I am now able to introduce my number system.

Symbols used: פ pronounced Pe (a capital letter of the Hebrew alphabet)

צ pronounced sadhe (a letter of the Hebrew alphabet)

Definition: Let פ be a metric space

פ = ({ℝ, ף(n, y, τ), all combinations of ף(n, y, τ) and the real numbers as defined addition, subtraction, multiplication and division*7} , צ)

Definition: Let the metric for פ be denoted by צ.

∀ $b, c \in$ פ and $b, c \in \mathbb{R}^+$ ∍ $|c(b,c)$ exists;

צ$(b,c) \equiv |c(b,c)$

צ(ף(n_0, y_0, τ), ף(n_1, y_1, τ)) = ף$(|n_1| + |n_0|, |y_1| + |y_0|, \tau)$
if $n_1 \geq 0,\ n_0 < 0$ or $n_1 < 0,\ n_0 \geq 0$

צ(ף(n_0, y_0, τ), ף(n_1, y_1, τ)) = $|n_1 - n_0| + |y_1 - y_0| \cdot \tau$
if $|n_1| \leq |n_0|,\ |y_1| \geq |y_0|$

צ(ף(n_0, y_0, τ), ף(n_1, y_1, τ)) = $|n_1 - n_0| - |y_1 - y_0| \cdot \tau$
for $|n_1 - n_0| - |y_1 - y_0| \cdot \tau \geq 0$

$= -(|n_1 - n_0| - |y_1 - y_0| \cdot \tau)$
for $|n_1 - n_0| - |y_1 - y_0| \cdot \tau < 0$

if $|n_1| \geq |n_0|,\ |y_1| \leq |y_0|$

} if $n_1 \geq 0,\ n_0 \geq 0$ or $n_1 < 0,\ n_0 < 0$

*8 Division by infinite numbers will be implicitly defined as the inverse of multiplication by infinite numbers.

Addition of two infinite numbers will be defined as follows:

$$\daleth(n_1, y_1, \tau) + \daleth(n_0, y_0, \tau) = \daleth(n_0 + n_1, y_0 + y_1, \tau)$$
$$\text{if } n_1 \geq 0,\ n_0 \geq 0 \quad \text{or } n_1 < 0,\ n_0 < 0$$

$$\daleth(n_1, y_1, \tau) + \daleth(n_0, y_0, \tau) = n_1 + n_0 + (y_1 + y_0)\cdot\tau$$
$$\text{if } n_1 \geq 0,\ n_0 < 0 \quad \text{or } n_1 < 0,\ n_0 \geq 0$$

Subtraction is defined via multiplication by negative one.

Multiplication of two infinite numbers is meaningless. I say this because I can think of no meaning to ascribe to it.

All operations not described here have been defined in my discussion of $\mathbb{R}^+$, and are identically defined for $\daleth$, (Recall that division by a real is the same as multiplication by one over that real.)

This completes the description of my number system. However, before I point out the principle conclusions that can be drawn from it there is an implied rule governing this system which I shall now state, explicitly, so that its importance not be underestimated.

RuleI: Whenever combining members of $\daleth$, the τ associated with each member of $\daleth$ must have the same value.

example 10 $\daleth(0,0,\tau_1) + 8 = \daleth(0,0,\tau_2)$

This is true, however, $\tau_1 \neq \tau_2$ therefore you can write

$8 = \daleth(0,0,\tau_2) - \daleth(0,0,\tau_1)$ but you cannot write $\daleth(0,0,\tau_2) - \daleth(0,0,\tau_1) = 0$

because $\tau_2 \neq \tau_1$ thus $\daleth(0,0,\tau_2) - \daleth(0,0,\tau_1) \neq 0$

example 11 $\dfrac{\frac{3}{4}}{\daleth(0,0,\tau)} = \dfrac{3}{4\cdot\daleth(0,0,\tau)} = \dfrac{3}{\daleth(0,0,\tau)}$

therefore $\frac{3}{4} = \frac{3}{\daleth(0,0,\tau)}\cdot\daleth(0,0,\tau)$ but $\frac{3}{4} \neq 3$

The reason for the inequality is that the τ associated with $\daleth(0,0,\tau)$ in the mumerator is of a different value than the τ as ociated with the $\daleth(0,0,\tau)$ in the denominator, hence the $\daleth(0,0,\tau)$'s cannot be cancelled.

Rule II describes the mechanics of rule one.

Rule II: Whenever $\gamma(0,0,\tau)$ is changed by any amount b , the entire real number system must be changed by the same amount b .

At present it is obvious that my number system contains infinite numbers, real numbers, and a largest and smallest real number, $\gamma(0,0,\tau)$ and $\gamma(-0,-0,\tau)$ respectivly. It, however, also contains real numbers which are infinitesmals.

Theorem: Given two sets A & B such that

$$A = \left\{ s \mid \forall b \in \{\mathbb{R} - \gamma(0,0,\tau)\}\ s = \frac{b}{\gamma(0,0,\tau)} \right\}$$

$$B = \left\{ d \mid d \in \{\mathbb{R} - 0\},\ d \notin A \right\}$$

then $\forall d \in B$ and $\forall s \in A$ $|s| \leq |d|$

Proof: Let $d \in B,\ s \in A$ Assume $|s| > |d|$

therefore $\left|\frac{b}{\gamma(0,0,\tau)}\right| > |d|$

Without loss of generality we can let $\frac{b}{\gamma(0,0,\tau)} > 0,\ d > 0$

therefore $\left|\frac{b}{\gamma(0,0,\tau)}\right| > |d| \iff \frac{b}{\gamma(0,0,\tau)} > d.$

$$\Rightarrow \frac{\frac{b}{d}}{\gamma(0,0,\tau)} > 1 \Rightarrow \frac{b}{d} > \gamma(0,0,\tau)$$

However this is false, hence a contradiction, hence $|s| \leq |d|$.

The reason this is false is because $s \in A$ hence $b \in \{\mathbb{R} - \gamma(0,0,\tau)\}$ $\Rightarrow b \neq \gamma(0,0,\tau) \Rightarrow b < \gamma(0,0,\tau)$; and since $d \in B$ $d \in \{\mathbb{R} - 0\}$ and $\forall a \in \mathbb{R}$ $d \neq \frac{a}{\gamma(0,0,\tau)} \Rightarrow \frac{b}{d} \neq \gamma(0,0,\tau)$ and since $\frac{b}{d} \in \mathbb{R} \Rightarrow \frac{b}{d} < \gamma(0,0,\tau)$ Q.E.D.

Corollary: $\forall a, b \in \{\mathbb{R} - \gamma(0,0,\tau)\}$

$$\frac{a}{\gamma(0,0,\tau)} < \frac{b}{\gamma(0,0,\tau)} \text{ iff } a < b$$

Proof: Given $a < b \Rightarrow \frac{a}{\gamma(0,0,\tau)} < \frac{b}{\gamma(0,0,\tau)}$

Given $\frac{a}{\gamma(0,0,\tau)} < \frac{b}{\gamma(0,0,\tau)}$ multiply by $\gamma(0,0,\tau) \Rightarrow a < b$

Q.E.D.

Definition: $\forall\, b \in \mathbb{R}$ and $\forall y \neq 0 \quad \dfrac{b}{\gamma(n,y,\tau)} \equiv 0$

Corollary: $\forall\, b \in \{\mathbb{R}-0\} \quad \exists\, c \in \{\mathbb{R}-0\} \ni |c| < |b|$

Proof: By the theorem just proven if there was a b for which no c existed it would be of the form $\dfrac{b}{\gamma(0,0,\tau)}$ and by the corollary to that theorem there is no smallest absolute value number of the form $\dfrac{b}{\gamma(0,0,\tau)}$, $b \neq 0$ Q.E.D.

Definition: $\forall y_1 \neq 0,\ y_0 \neq 0 \quad \dfrac{\gamma(n_1,y_1,\tau)}{\gamma(n_0,y_0,\tau)} = \dfrac{n_1}{n_0} + \left(\dfrac{y_1}{y_0}\right)\cdot\tau$

Corollary: $\forall b \in \{\daleth - 0\} \quad \exists\, c \in \{\daleth - 0\} \ni |c| < |b|$

Proof: $\daleth - \mathbb{R}$ = all infinite numbers.

therefore saying $\forall b \in \{\daleth-0\}\ \exists\, c \in \{\daleth-0\} \ni |c| < |b|$
$\Longleftrightarrow$ saying $\forall b \in \{\mathbb{R}-0\}\ \exists c \in \{\mathbb{R}-0\} \ni |c| < |b|$ and
$\forall b = \gamma(n,y,\tau)\ \ y \neq 0 \ \ \exists\, c \in \{\daleth - 0\} \ni |c| < |b|$

And this is true because I have already proven the first part and the part is true if $c \in \{\mathbb{R}-0\}$. Q.E.D.

It should be obvious from the theorem just proven that any real number of the form $\dfrac{b}{\gamma(0,0,\tau)}$, $b \neq \gamma(0,0,\tau)$, $b \in \{\mathbb{R}-0\}$ is an infinitesmal.

Therefore the primary advantage and beauty of my number system is that whenever infinitesmals or infinite numbers must be dealt with, they are in the number system and can be used with the same ease and understanding as a real number. However, when using infinitesmals and infinite numbers you must be sure to obey Rule I. For example if one was trying to solve the equation $y + 10 = \dfrac{y}{\gamma(0,0,\tau)}$ you could multiply both sides by $\gamma(0,0,\tau)$ and the equation would become $y\,(\gamma(0,0,\tau)) + 10(\gamma(0,0,\tau)) = y$ or $\gamma(0,0,\tau_1) + \gamma(0,0,\tau_2) = y$, $\tau_1 \neq \tau_2$, in order to continue you must

make $\tau_1 = \tau_2$. To do this you would have to multiply $\eta(0,0,\tau_1)$ by 10 and $\eta(0,0,\tau_2)$ by y , this, however, would destroy the equality, hence this equation is not solvable by ordinary means. The method I use to solve such equations is the trial and error method.*8 Another phenomenon of my number system is $0 \times (0 \times \eta(n,y,\tau)) = 0 \times \eta(0,0,\tau) = 0 \quad \forall y \neq 0$
but $(0 \times 0) \times \eta(n,y\,\tau) = 0 \times \eta(n,y,\tau) = \eta(0,0,\tau) \quad \forall y \neq 0$
therefore, $(0 \times 0) \times \eta(n,y,\tau) \neq 0 \times (0 \times \eta(n,y,\tau)) \quad \forall y \neq 0$
Hence you must bear these in mind when using this number system.

The final two advantages of my number system deal with its treatment of zero and infinite cardinalities. In the number system I have just described division by zero no longer has mysterious undertones, such as those illustrated by the expression $\lim^{+}_{x \to 0} \frac{1}{x} = \infty$ What this tells us is that the function $\frac{1}{x}$ is real valued until $x = 0$, at which point the function jumps off the real number line and goes to a mysterious place called infinity. Then as soon as $x < 0$ the function comes back from this place called infinity and is real valued again. What my system tells us is that $\lim^{+}_{x \to 0} \frac{1}{x} = \eta(0,0,\tau)$

There are, of course , many other results of modern mathematics that will have to be reanalized using my number system, I however, shall only deal with only one of these, the assignment of infinite cardinalities to infinite sets.*9

*8 An easy equation to solve is $y = \dfrac{y}{\eta(0,0,\tau)}$, the solution is $y = 0$

*9 When I origionally undertook to write this paper, it was with the intention of finding a method by which infinite cardinalities might be assigned to infinite sets. For, once I had familiarized myself with Cantor's method, I was determined to find another one.

From its name, an infinite set is one that has an infinite number of members, therefore its cardinal number must be a positive infinite number. A positive infinite number is represented by a point which is initially beyond the right end of the real number line, and is moving away from the right end of the real number line at some speed y . The question, is how to associate an infinite number to an infinite set.

Given the infinite set $I = \{1, 2, 3 \ldots\}$ and given the infinite numbers $\daleth(1,1,\tau)$, $\daleth(3,8,\tau)$, $\daleth$, $\daleth(4,6,\tau)$, $\daleth(10,10,\tau)$, etc. There is no apriory way to associate one of these infinite numbers with the set I . On the other hand, there is no reason why any one of these infinite numbers could not represent the cardinality of I ; for they are all positive infinite numbers and the only requirement for the cardinal number of I is that it be a positive infinite number. Thus we come to the conclusion that any randomly choosen infinite number could represent the cardinality of I .

Therefore given any infinite set A the only restriction placed on the cardinal number of A is that it be infinite and positive, hence any number $\daleth(n,y,\tau) \ni y > 0$ may represent the cardinal number of A . If, however, we are given a second infinite set, B , we must determine whether the existance of A places any restrictions on the cardinal number of B.

There are seven possible cases: $\subset$ symbolizes, is a proper subset of.

Case 1: $B = A$

Case 2: $B \subset A$, a one to one correspondence can be established between the elements of A and B .

Case 3: $B \subset A$, a one to one correspondence cannot be established between the elements of A and B .

Case 4: $B \supset A$, a one to one correspondence can be established between the elements of A and B.

Case 5: $B \supset A$, a one to one correspondence cannot be established between the elements of A and B.

Case 6: $B \not\supset A$, $B \not\subset A$, $B \neq A$ a one to one correspondence can be established between the elements of A and B.

Case 7: $B \not\supset A$, $B \not\subset A$, $B \neq A$ a one to one correspondence cannot be established between the elements of A and B.

If $\eta(n_0, y_0, \tau)$ is the cardinal number of A then in cases 1,2,4,and 6 $\eta(n_0, y_0, \tau)$ is the cardinal number of B.

If we say that the cardinal number of B is $\eta(n_1, y_1, \tau)$ then in case 3 $\eta(n_1, y_1, \tau) \not> \eta(n_0, y_0, \tau)$ therefore $\eta(n_1, y_1, \tau) \leq \eta(n_0, y_0, \tau)$ in case 5 $\eta(n_1, y_1, \tau) \not< \eta(n_0, y_0, \tau)$ therefore $\eta(n_1, y_1, \tau) \geq \eta(n_0, y_0, \tau)$ in case 7 $\eta(n_1, y_1, \tau)$ is not related to $\eta(n_0, y_0, \tau)$.

Thus given any infinite set A and any other infinite set B; if the cardinal number of A is $\eta(n_0, y_0, \tau)$ then B may also have $\eta(n_0, y_0, \tau)$ as its cardinal number. Hence any one infinite number may represent the cardinal number of all infinite sets.

Theorem: Given $A = \{1, 2, 3, \ldots\}$ there is no infinite set B that contains less elements then A.

Proof: If the elements of A are corresponded to the elements of B such that each element of A is corresponded to a different element of B and the elements of A are used in the order 1,2, 3, etc. then if there are less elements in B then in A, there will be some $a_0 \in A \ni \forall a_1 \in A$, $a_1 > a_0$ there is no element of B. Therefore, there are a_0 elements in B, but since $a_0 \in A$, a_0 is finite, therefore B is finite. $\times$ Thus we conclude if B is infinite there are no less elements in B then in A.

Theorem: If $A = \{1, 2, 3, \ldots\}$ and the cardinal number of A is choosen to be פ , then all infinite sets B , whether $B = A$, $B \neq A$ have פ as their cardinal number.

Proof: Cases 1,2,4, and 6 are obvious. By the last theorem, we see that no infinite set B can have a smaller infinite number, and since there is no larger infinite number then פ , in cases 3,5, and 7, the cardinal number of B must also be פ . Since all infinite sets B must fall into one of the seven cases, all infinite sets B , whether $B = A$ or $B \neq A$ have cardinal number פ .

That all infinite sets can have the same cardinal number tells us that all infinite sets have the same number of elements. The fact that some infinite sets can have different cardinal numbers does not detract at all from the conclusion that all infinite sets have the same number of elements. The reason for this is that all infinite sets have an infinite number of elements, therefore their cardinal number is an infinite number, and the size of an infinite number is constantly increasing (ie. its distance from its end of the real number line is constantly increasing) : hence all infinite numbers can have the same size, but at different τ's .

Read appendix II.

What can be concluded from this paper is that there is a number system, the Pe (ף) number system, which has, real, infinite, and infinitesmal real numbers, in it. Thus all mathematics employing the concepts of infinity and, or, infinitesmals, that was previously done, must be redone using my number system, and any mathematics to be done with these concepts must be done using my number system.

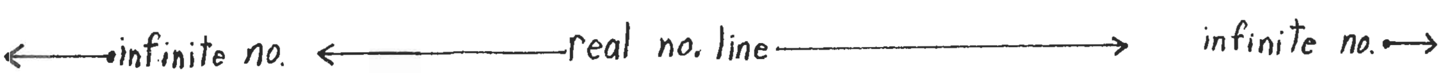
Appendix I

Philosohyp Behind the Lamed's

(Note: the Lamed's are my infinite numbers)

If I say that the real number line, at this instant, contains all the real numbers, and I then ask the question, how long is this line? You would answer by saying that it is indefinitely long. I now ask, can I create new real numbers that are not on this line? The obvious answer is no. I, however, would say that if there are no fixed boundries on this real number line then I can expand this line and hence make new real numbers. If you ask, why can I expand the real number line? I would answer, if there are no fixed bounds, why can't I expand it. And I am sure you will agree that there is no fixed greatest real number, thus there are no fixed bounds on the real number line.

Hence we have come to the realization that a real number line containing "all" the real numbers and extending out indefinitely can be expanded so that it will contain more (new) real numbers. I now ask myself, what is the significance of this development. The answer, is that it leads to a new picture of the real number line and infinite numbers. This picture of the real number line (my Picture) is as follows.) The real number line is an indefinitely great line which has end points, these end points, however, are constantly moving away from the zero point of the real number line.[*1] An infinite number is a number which is initially beyond an end point of the real number line and is moving away from that end of the real number line with some speed.



Therefore when you think of the real number line think of it as an indefinitely great line that is constantly expanding. When you think of

*1 This is my definition of the real number line.

an infinite number think of it as a point somewhere beyond one of the end points of the real number line and moving away from this end point.

Appendix II

The purposes of this appendix are to explain certain points concerning the Pe (⁊) number system, which, although covered in my paper may not be obvious; and to elaborate on my analysis of infinite cardinals.

When I defined zero divided by zero to equal zero ($\frac{0}{0} \equiv 0$) I did this because it was the only definition that did not cause contradictions.

The term size of an infinite number has been used in my paper, it of course, refers to the value obtained from the expression ʒ $(7(n,y,\tau),0)$ where $7(n,y,\tau)$ is the infinite number in question.

Given two infinite numbers $7(n_1,y_1,\tau)$ & $7(n_2,y_2,\tau)$ the following is true $7(n_1,y_1,\tau) - 7(n_2,y_2,\tau) = b$ where $b \in$ ⁊ and

$$7(n_1,y_1,\tau) = 7(n_2,y_2,\tau) \quad \text{if} \quad b=0$$
$$7(n_1,y_1,\tau) > 7(n_2,y_2,\tau) \quad \text{if} \quad b>0$$
$$7(n_1,y_1,\tau) < 7(n_2,y_2,\tau) \quad \text{if} \quad b<0$$

Two or more different infinite numbers can have the same value at different τ's . For example given $7(1,1,\tau_1)$ & $7(3,8,\tau_2)$ if $\tau_1 = 10$ then $7(1,1,\tau_1)$ is $\tau_1 + 1 = 11$ units away from the right end of the real number line, at $\tau_1 = 10$. And if $\tau_2 = 2$ then $7(3,8,\tau_2)$ is $\tau_2 \cdot 8 + 3 = 11$ units away from the right end of the real number line, hence $7(3,8,\tau_2)$ has the same value as $7(1,1,\tau_1)$ if $\tau_1 = 10$ and $\tau_2 = 1$.

In my explanation of infinite cardinals I stated that all infinite sets could have the same infinite cardinal number. I then stated that infinite sets could have different infinite cardinal numbers. The reason for this comes out of my analysis of the seven cases. In case 3 if $7(n_0,y_0,\tau)$ is the cardinal number of the infinite set A and $7(n_1,y_1,\tau)$ is the cardinal number of the infinite set B , then $7(n_1,y_1,\tau) \leq 7(n_0,y_0,\tau)$ What this says is that $7(n_1,y_1,\tau)$ may be equal to $7(n_0,y_0,\tau)$, or, $\eta(n_1,y_1,\tau)$ may be less then $\eta(n_0,y_0,\tau)$. Either choice of values for $\eta(n_1,y_1,\tau)$ is valid. A similar statement can be made in regards to case 5. Therefore barring restrictive theorems different infinite sets may have different cardinal numbers or the same cardinal number.*1

The number $\eta(0,0,\tau)$ has certain properties which are not obvious.

$I = \{0, 1, 2, 3, \ldots\}$ Q = set of rational positive numbers

IR = set of positive irrational numbers

$\mathbb{R}$ = set of positive rational numbers

Theorem A : $\forall r \in \mathbb{R} \;\; \exists n \in I \;\; \ni \;\; n > r$

Proof: There are three cases

Case1 $r \in I$ let $n = r+1$ $\therefore n > r$

Case2 $r \in Q$, $r \notin I$ $\Longrightarrow r = \frac{a}{b}$, $a, b \in I$, $b \neq 1, 0$ let $n = a$ $\therefore n > r$

Case3 $r \in IR$ since all real numbers can be written as infinite descimals we have $r = a_0 b_1 b_2 b_3 \ldots$ where $a_0, b_1, b_2, b_3 \ldots \in I$ let $n = a + 2$ therefore $n > r$

Q.E.D.

Note however, that $\eta(0,0,\tau)$ cannot represent the cardinality of the set of integers because it does not represent the total number of integers that can exist. There are an infinite number of integers that can exist, hence the set of integers has an infinite cardinality.

It can also be shown that there is an irrational number b larger then n , namely $b = n + \sqrt{2}$ and there is a rational number not an integer c larger then n, namely $c = n + \frac{1}{2}$. What this tells us is that $\eta(0,0,\tau)$ is real and not an integer or rational, it is rational and not an integer, it is an integer. Obviously it cannot be all these things at the same instant. And it isn't, as stated in appendix II the end of the real number line is constantly expanding, or moving, thus constantly

*1 My last theorem is an example of one type of restriction.

forming new real numbers. Thus we can see that at different τ's $\daleth(0,0,\tau)$ is becoming different real numbers and that $\daleth(0,0,\tau)$ will be real numbers of all three catagories, irrationals, rationals, integers.

Let us consider the question $a^{\daleth(n,y,\tau)} = ?$

Definition: $\forall a \in \mathbb{R} \quad \exists\ \daleth(n_1, y_1, \tau) \in \daleth \quad \ni \quad a^{\daleth(n,y,\tau)} = \daleth(n_1, y_1, \tau)$

This does not imply that $a \cdot a \cdot a \cdots = \daleth(n_1, y_1, \tau)$

Just as $2 \cdot \daleth(n, y, \tau) = \daleth(2n, 2y, \tau) \not\Rightarrow 2 \cdot 2 \cdot 2 \cdots = \daleth(2n, 2y, \tau)$

Definition: $\forall\ \daleth(n_1, y_1, \tau) \in \daleth \quad \exists\, a \in \mathbb{R}$

$\ni \quad \daleth(n_1, y_1, \tau)^0 = a$

Note: In the above difinitions unless there is a reason why the a's n's or y's should take on a specific value they sahll be left as variables in all expressions containing $a^{\daleth(n,y,\tau)}$ or $\daleth(n_1, y_1, \tau)^0$.

Definition $\forall a \in \mathbb{R} \quad a^{\daleth(0,0,\tau)} = \daleth(0,0,\tau)$

Recall that $\daleth(0,0,\tau) \in \mathbb{R} \quad \therefore \daleth(0,0,\tau)^0 = 1$

Those familiar with Cantor's analysis of infinite cardinals may wonder at the different conclusions that I came up with. Obviously I cannot do justice to Cantor's work in a one page critisysm. Therefore I shall merely illustrate the oversights in his two main proofs, which invalidate them, and discuss the faulty reasoning that lead to these oversights.

Where $\mathcal{P}(A)$ stands for the power set of A. And $\overline{\overline{A}}$ means the cardinal number of A.

Cantor's Theorem: $\overline{\overline{A}} < \overline{\overline{\mathcal{P}(A)}}$

Cantor's Proof: Obviously $\overline{\overline{A}}$ is not greater then $\overline{\overline{\mathcal{P}(A)}}$. Therefore $\overline{\overline{A}} \leq \overline{\overline{\mathcal{P}(A)}}$ thus we assume $\overline{\overline{A}} = \overline{\overline{\mathcal{P}(A)}}$. Let $f: (A) \longrightarrow \mathcal{P}(A)$ show that these sets have the same number of elements; now consider $A_1 = \{a \in A \mid a \notin f(a)\}$

since A_1 is an element of $P(A)$ $\exists a_1 \in A \ni A_1 = f(a_1)$ but if $a_1 \in A \Rightarrow a_1 \notin f(a_1) \Rightarrow a_1 \notin A_1$ —x— or if $a_1 \notin A_1 \Rightarrow a_1 \in f(a_1) \Rightarrow a_1 \in A_1$ —x— .

However, in this proof the function mapping A into $P(A)$ is restricted to a function which satisfies the condition $\forall a \in A \;\; a \notin f(a)$ Therefore, what has been proven is that no function f exists $\ni \forall a \in A$ $a \notin f(a)$, he has not shown that no function g , with no restrictions on it, exists such that $g: A \rightarrow P(A)$.
Thus in general this theorem has not been proven, it has only been proven under restrictive circumstances, therefore it is generally invalid.*2

The same type of mistake is also found in the famous Cantor diagonal process, by which he proves that there are less integers then reals. He states the following; all real numbers can be written as infinite decimals, therefore if there are the same number of reals as integers I can set up the following one to one correspondence between the reals and the integers. Where the a_{ij} are all integers.

$1 \leftrightarrow a_1 . a_{11} a_{12} a_{13} a_{14} \ldots$
$2 \leftrightarrow a_2 . a_{21} a_{22} a_{23} a_{24} \ldots$
$3 \leftrightarrow a_3 . a_{31} a_{32} a_{33} a_{34} \ldots$
etc.

However, I can create a new real, not in this correspondence, as follows; where the b_k are all integers the new real is $b . b_1 b_2 b_3 b_4 \ldots$ where $b_k \neq a_{kk}$. Therefore there is a real left over so there are more reals then integers. What he has failed to realize is that the fact that there is a real not in the correspondence means one of two things; either there are more reals then integers, or he has set up the wrong correspondence between the reals and integers. For example, I can set up the following correspondence: Where a_{ij} are all integers.

$1 \leftrightarrow a_1 . a_{11} a_{12} a_{13} a_{14} \ldots$
$101 \leftrightarrow a_2 . a_{21} a_{22} a_{23} a_{24} \ldots$

*2 Without this theorem Cantor's paradox also dissapears.

$201 \leftrightarrow a_3 . a_{31} a_{32} a_{33} a_{34} \ldots$

$301 \leftrightarrow a_4 . a_{41} a_{42} a_{43} a_{44} \ldots$

etc.

In this correspondence you now have many integers as well as reals left over, which proves absolutely nothing, just as Cantor's diagonal process proves absolutely nothing.

The reason for these errors lie in Cantor's method of analysing infinite cardinals. Firstly he does not determine what an infinite number is. He merely lets the symbol $\aleph$ (the Hebrew letter capital aleph) stand for an infinite number, and then determines properties of the symbol $\aleph$ and assumes these properties will also be properties of infinite numbers. The false conclusion he arrived at was that the cardinal number of $\mathbb{R}$ is greater then the cardinal number of I, where $\mathbb{R}$ = set of reals, I = set of integers. What this says is that you will run out of elements of I before you will run out of elements of $\mathbb{R}$, however, both $\mathbb{R}$ and I are infinite, thus you never run out of elements of either set, therefore, saying the number of elements of $\mathbb{R}$ is greater then the number of elements in I is nonsense.

Finally, although I have stated that $\daleth$ is a metric space with metric $\mathfrak{d}$, I have not formally proven this. Since $\mathbb{R}$ is a metric space the only thing to prove is that $\forall a \in \mathbb{R}$, $\forall \daleth(n, y, \tau) \in \daleth$

$\mathfrak{d}(\daleth(n, y, \tau), a)$ has the metric space properties

$\mathfrak{d}(\daleth(n_1, y_1, \tau), \daleth(n_2, y_2, \tau))$ has the metric space properties

The only metric space property which is not obvious is the triangle inequality, and this can be proven by anyone wishing to take the time to expand each case seperately.